Date 11/12/20

Numerical Analysis

Part III

Newton - RAPHSON METHOD

Let x_0 be an approximate value of a root of the equation $f(x) = 0$ and let $x_0 + h$ be the exact value of the corresponding root, where h is very small quantity, positive or negative.

Then $f(x_0 + h) = 0$

Since $x_0 + h$ is the root of the equation $f(x) = 0$

expanding (1) by Taylor's Theorem, we get

$$f(x_0 + h) = f(x_0) + h f'(x_0) + \frac{h^2}{2!} f''(x_0) + \text{——} = 0$$

Since h is very small, we may neglect second and higher order terms.

Hence as the first approximation we have

$f(x_0) + h f'(x_0) = 0$

$$\Rightarrow h = -\frac{f(x_0)}{f'(x_0)}$$ Provided

$$f'(x_0) \neq 0$$

Hence $$x_1 = x_0 + h = x_0 - \frac{f(x_0)}{f'(x_0)} \quad \text{—} (2)$$

The equation (2) gives the improved value of the root over the previous one

Now substituting x_1 for x_0 and x_2 for x_1 in (2); the second approximation x_2 is obtained by the equation

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_2)} \quad \text{—} (3)$$

In General, we can get an approximation on the iteration formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{—} (4)$$

From this formula, we can calculate successive better values of the root

The formula (4) is known as Newton-Raphson method.